

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

8 April 2025

Subject CM1 – Actuarial Mathematics for Modelling Core Practices

Paper A

Time allowed: Three hours and twenty minutes

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.
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1 Calculate $A_{[70]:\overline{3}|}$.

Basis:

Mortality: AM92 Select

Interest: 5% p.a. effective

[3]

2 An investor purchases an 8-year, fixed-interest bond at issue. The bond pays half-yearly coupons in arrears at a rate of 3% p.a. and will be redeemed at 104%.

The investor is subject to income tax at 20% and capital gains tax at 25%.

The bond is priced to achieve a net effective real yield of 6.5% p.a.

Calculate, showing all working, the price per \$100 of the bond at issue. [5]

3 An annuity provides a benefit of £20,000 p.a. payable annually in advance throughout the life of the policyholder aged 50 exact.

Calculate, showing all working:

(a) the expected present value of the benefits.

(b) the variance of the present value of the benefits.

Basis:

Mortality: AM92 Select

Interest: 4% p.a. effective

[4]

- 4** The force of interest per annum at time t years is given by:

$$\delta(t) = \begin{cases} 0.05 & 0 < t \leq 6 \\ 0.026 + 0.004t & 6 < t \leq 12 \end{cases}$$

£1,500 is invested at $t = 1$ year for a period of 5 years.

- (i) Calculate, showing all working, the accumulated value of this investment at the end of the 5-year period. [1]
- (ii) Calculate, showing all working, $i^{(12)}$, the annual nominal rate of interest convertible monthly, earned on the investment.

You should express your answer as a percentage rounded to the nearest three decimal places. [2]

A continuous payment stream is received at a rate of $e^{0.02t}$ units per annum between $t = 2$ and $t = 6$.

- (iii) Calculate, showing all working, the accumulated value of the payment stream at $t = 10$. [5]
- [Total 8]

- 5** An annuity of £30,000 p.a. is payable monthly in arrears for a period of 10 years. The annuity is deferred for exactly 5 years.

Calculate, showing all working and using appropriate annuity functions, the present value of the deferred annuity, assuming that the following interest rates apply for the 15-year period:

- 4% p.a. effective for the first 8 years
- 6% p.a. nominal convertible monthly for the remaining 7 years.

[7]

- 6 A life office issues a unit-linked endowment policy with a term of 3 years to a life aged 55 exact.

Details of the policy are as follows:

- The premium is £1,800 payable annually in advance.
- The death benefit, payable at the end of the year of death and after the Annual Management Charge (AMC) has been deducted, is the greater of £2,500 and the bid value of the units.
- The maturity value is equal to the bid value of the units.
- The unit fund cashflows (all figures in £) are:

	<i>Year</i>		
	<i>1</i>	<i>2</i>	<i>3</i>
Unit fund at start of year	0.00	1,730.44	3,535.72
Allocated premium	1,710.00	1,710.00	1,710.00
Bid-offer spread	−51.30	−51.30	−51.30
Interest	116.11	237.24	363.61
Unit fund at end of year (before AMC)	1,774.81	3,626.38	5,558.03
AMC	−44.37	−90.66	−138.95
Unit fund at end of year (after AMC)	1,730.44	3,535.72	5,419.08

Calculate, showing all working:

- (a) the profit signature.
- (b) the expected present value of the profits.

Pricing basis:

Mortality: AM92 Ultimate

Rate of interest earned

on non-unit cash flows: 5% p.a. effective

Expenses: £200 at the start of each policy year

Risk discount rate: 10% p.a. effective

[7]

- 7 A 20-year loan of \$250,000 is repaid by level monthly repayments in arrears. The effective rate of interest applicable to the loan is 6% p.a. effective for the first 5 years and then 7.5% p.a. effective for the rest of the term.

- (i) Calculate, showing all working and using appropriate annuity factors, the monthly repayment. [4]
- (ii) Calculate, showing all working and using appropriate annuity factors, the capital and interest elements of the 61st instalment. [3]
- (iii) Calculate, showing all working, the total interest paid on the loan. [2]

[Total 9]

- 8** A special joint whole of life policy is issued to a male life aged 54 exact and a female life aged 51 exact. The policy pays £20,000 immediately on the second death.

Level premiums are payable monthly in advance ceasing after 10 years, or on the first death if earlier.

If both lives survive for 15 years from policy inception, a lump sum benefit equal to a full year of premiums is immediately payable.

Calculate, showing all working, the monthly premium payable.

Basis:

Interest rate: 4% p.a. effective

Mortality: PMA92C20 for the male and PFA92C20 for the female

Expenses: None

[9]

- 9** A fixed-interest bond pays annual coupons in arrears of 6% p.a. and is redeemable at 103% in 3 years' time.

The following forward rates apply:

	<i>Time, t (years)</i>		
	0	1	2
f_t (%)	2.2	2.4	2.7

f_t is the 1-year discrete forward rate of interest over the year from time t to time $t + 1$.

- (i) Calculate, showing all working:
- (a) the price per \$100 nominal of the bond.
 - (b) the 3-year par yield.
 - (c) the gross redemption yield of the bond, using linear interpolation.

[7]

- (ii) Explain how your answers to parts (i)(b) and (i)(c) would be affected, if at all, by an increase in the coupon rate.

[3]

[Total 10]

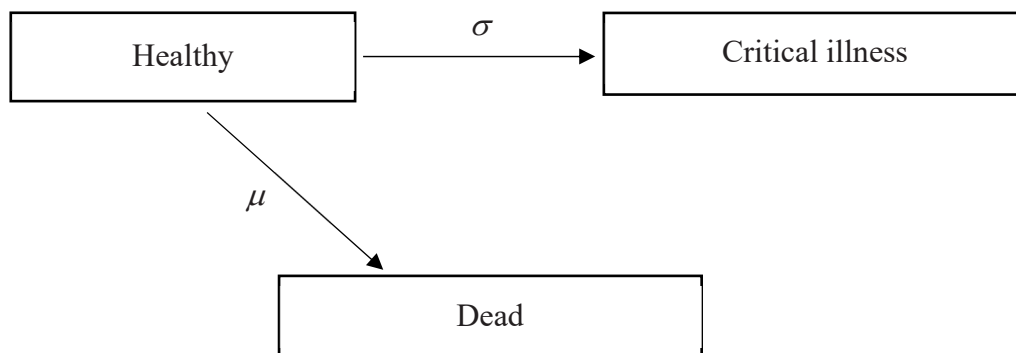
- 10** A life insurance company issues 30-year term critical illness policies to lives aged 35 exact.

The benefit is a lump sum payable immediately on diagnosis of a critical illness as follows:

- No critical illness benefit is payable in the first 2 years of the policy.
- £100,000 if the critical illness is diagnosed between 2 and 10 years after issue.
- £150,000 if the critical illness is diagnosed between 10 and 30 years after issue.

Level premiums are payable continuously throughout the term from the date of issue and cease immediately on a claim.

The insurance company uses the following three-state continuous-time Markov model to calculate the premium and reserves for the policy.



- (i) Calculate, showing all working, the annual premium payable continuously. [8]
- (ii) Calculate, showing all working, the prospective gross premium reserve after 11 years. [3]

Basis:

Mortality: $\mu = 0.005$

Critical illness: $\sigma = 0.002$

Force of interest: 6% p.a.

[Total 11]

- 11** On 1 January 2019 a life insurance company issued increasing whole life assurance policies to single lives aged 55 exact.

The death benefit, payable at the end of the year of death was £50,000 in the first policy year, thereafter increasing at the beginning of each subsequent policy year at a rate of 1.92308% p.a. compound.

The level annual premiums are payable annually in advance throughout the policyholder's life.

- (i) Show that the premium is approximately £1,463. [3]

On 1 January 2024 there are 500 policies still in force. During 2024, 6 policyholders die.

- (ii) Calculate, showing all working, the mortality profit or loss for the calendar year 2024. [7]

- (iii) Comment on your answer to part (ii). [2]

Basis:

Mortality: AM92 Ultimate

Interest: 6% p.a. effective

Expenses: None

[Total 12]

- 12** A life insurance company issues a 25-year with profit endowment assurance policy. The benefit is the basic sum assured plus any attaching bonus and is payable at the end of the year of death or on survival to the end of the term.

Level premiums are payable monthly in advance ceasing after 25 years or on the death of the policyholder if earlier.

Simple reversionary bonuses vest at the end of each policy year (i.e. the death benefit does not include any bonus relating to the policy year of death).

The company issues the policy to lives aged 40 exact. The basic sum assured is \$250,000.

- (i) Calculate, showing all working, the monthly gross premium for this policy. [12]

For the first 15 years of the policy, the actual bonuses declared were equal to those assumed when calculating the gross premium.

- (ii) Write down the equation for the gross premium prospective reserve at the end of the 15th policy year. You are not required to perform any calculations. [3]

Basis:

Mortality: AM92 Ultimate

Interest rate: 6% p.a.

Commission:

Initial: 15% of the total premiums payable in the first policy year

Renewal: 2.5% of the second and subsequent monthly premiums

Expenses:

Renewal: \$80 at the start of each policy year, excluding the first

Bonus: Simple bonus rate of 1% of basic sum assured

[Total 15]

END OF PAPER